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Investigating Pedagogical Content Knowledge and Decision-Making on Algebraic Tasks

Ajeng Gelora Mastuti

Institut Agama Islam Negeri Ambon
Corresponding Author: ajeng.gelora.mastuti@iainambon.ac.id

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ABSTRACT

Pedagogical Content Knowledge (PCK) tasks and decision-making are closely related to teaching and learning mathematics. This study aims to investigate teacher PCK in decision-making on different algebra tasks and to see activity in teacher-student interactions. This research is qualitative, with a research approach used as a case study. The participants in this study were only one teacher with different characteristics in mathematics tasks. Data collection consisted of primary research data from class observations, teacher interview transcripts, and researcher notes, while documents such as teacher notes were secondary data. Data analysis was carried out directly based on class observations and interviews. The findings in this study reveal teacher PCK through teacher decision-making regarding algebraic tasks in Swartz & Perkins (2016) three theories of thinking: generating ideas, clarifying ideas, and assessing the reasonableness of ideas.

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INTRODUCTION

Tasks are essential in mathematics class (Berisha & Bytyqi, 2020; Demosthenous et al., 2021; Henningsen & Stein, 1997; Mastuti & Prayitno, 2023). Tasks are central to teaching (Chapman, 2013; Papadopoulos, 2020). Mathematics tasks affect student learning in mathematics classes, so teachers must choose meaningful mathematics tasks (Henningsen & Stein, 1997; Johnson et al., 2017). In addition, mathematics tasks affect students' thinking and understanding of mathematics. Mastuti et al. (2022) researched teachers who successfully selected and prepared types of mathematics tasks that resulted in high student learning outcomes. Therefore, in the 21st century mathematics teachers are needed who have the 4Cs (critical thinking, creative thinking, collaboration, and communication) skills.

Pedagogical Content Knowledge (PCK) tasks and decision-making are closely related

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to teaching and learning mathematics (Fukaya et al., 2022; Yurniwati & Kustandi, 2022). One of the six principles of effective teaching, according to Ormond (2021), is to involve students by utilizing a variety of rich and challenging tasks and giving students time and opportunities to make decisions and use various forms of representation. Therefore, tasks should ask students to justify, direct students to understand mathematical ideas used to solve problems, encourage students to make connections between strategies, and encourage students to formulate and prove conjectures and generalizations (Mastuti, Abdillah, & Rijal, 2022; Stylianou, 2013). Therefore, the selection and appearance of different tasks are challenging for mathematics teachers (Abdillah & Mastuti, 2018; Bringula et al., 2021).

In Mastuti & Prayitno (2023) preliminary study on the exploration of algebra-rich task designs for high school teachers, the authors observed the mathematics teacher's tasks in a different class, namely class X, in 5 schools in East Java, namely SMAN 1 Bangsal, MAN 2 Malang, SMAN 3 Jombang, SMA Hang Tuah Surabaya, and SMAN 1 Puri. However, from several class observations, a teacher is interested in giving tasks. In addition, the teacher asks interesting questions on algebra material. For example, teachers rarely ask, "What was the result?" and "How many solution sets?" However, the enthusiastic response of the students made the class lively and lively with each student's comments.

When selecting tasks, the first decision for teachers is to identify students' potential tasks and to what extent it fits their curriculum goals. Appreciation of the inherent mathematical ideas and pedagogical actions required to implement tasks (Kim et al., 2019; Sullivan et al., 2015). Connected with this is identifying the tasks' purpose to inform their interactions with students and articulate that purpose to students.

These phenomena are directed by Henningsen & Stein (1997) regarding PCK, which includes pedagogical knowledge about tasks which are also very important here. Mathematical tasks can also be used to access problems related to mathematical pedagogy (Bringula et al., 2021; Li & Schoenfeld, 2019). Mathematics tasks should provide a context for exploring issues such as the nature of mathematics, assessment, constructivism, social constructivism, group work, and so on. Therefore, it is essential to provide the boundaries of the task. Teachers use some of the necessary considerations in making decisions about their knowledge. These considerations include: considering several options through brainstorming, predicting consequences, and making the 'best' choice by weighing the pros and cons (Darling-Hammond et al., 2020). The difference between this study and other studies lies in the teacher's decision-making involving different algebraic tasks and involving teacher-student interaction in their observations.

The decision-making referred to in this article is described using three basic categories: generating ideas, clarifying ideas, and assessing the fairness of ideas (Swartz & Perkins, 2016). The importance of decision-making skills in the first category is traditional creative thinking skills; the second is analytical thinking skills; and the third is critical thinking skills related to reasoned critical judgments. So, the purpose of this article is to investigate teacher PCK in decision-making on planned algebra tasks and to see activity in teacher-student interactions.

RESEARCH METHOD

This research is qualitative, with a research approach used as a case study. This study aims to describe teacher PCK in making decisions about algebra tasks and to describe student activities toward teacher algebra assignments. This study investigates teacher PCK based on teacher-student task interactions and teacher information. The PCK components used by the teacher in making decisions to develop different tasks consist of 1) knowledge of algebra material, 2) knowledge of students' conceptions and misconceptions, and 3) knowledge of task strategies. In this study, observations were made of class X SMA, video-based interviews, and in-depth audio recordings of the teachers who were the subject of this study.

Participants in this study were only one teacher with different characteristics in algebra tasks. Based on previous research by the author on Mastuti & Prayitno (2023), the author found a teacher who did excellent teaching in learning mathematics. Other supporting instruments in this study were learning observation sheets to observe PCK teachers and interview guidelines to explore teacher decision-making in mathematical tasks. Data was collected through interviews by combining structured and unstructured interviews. The data was taken to observe teacher PCK based on teacher-student activities. Table 1 below is an element of PCK observed in teacher-student interactions in learning.

Table 1. PCK Elements Observed in Teacher-Student Task Interactions

Knowledge of the Task Material	Knowledge of Task Strategy	Knowledge About Student Conception
• Demonstrate conceptual and procedural understanding to identify material aspects of teacher tasks. By checking: a. The Truth of mathematics facts b. Flexibility in explanations c. Sequential representation of facts d. Hierarchical presentation e. Easy to understand the idea	• Using understandable instructions in tasks. • Use examples and analogies in instructions. (Provoking students' prior knowledge by asking questions to connect to previous knowledge) • Present strategies that make students comfortable	• Understand students' misconceptions about instructions in tasks and students' answers. • Knowing the possibility of student difficulties during tasks. • Gives a wide space to the way students think about concepts

Knowledge of the Task Material	Knowledge of Task Strategy	Knowledge About Student Conception
• Critically identify mathematical material in each task concept (you can also explore student understanding by giving additional questions spontaneously to complete teacher tasks) to increase students' understanding of concepts. • Make clarifications to display skills in solving problems in their tasks.	presenting their arguments in each task.	• Be fully aware of the tasks given to students to measure learning.

Data collection consisted of primary research data from class observations, teacher interview transcripts, and researcher notes, while documents such as teacher notes were secondary data. Researchers use several recording devices, such as Handycams, voice recorders, and notes, to record learning involving class tasks. After each lesson, the researcher immediately clarified the subject. Then, before analyzing the data, the researcher triangulated the data. The data triangulation used is a triangulation of data sources, which aims to dig up certain information truths through various methods and sources of data acquisition on direct observation data, interviews, and the collection of physical evidence such as tasks in the previous material.

Data analysis was carried out directly based on class observations and interviews. Researchers observed, collected data, and conducted in-depth interviews with mathematics teachers in every process of learning mathematics. Interviews are based on questions asked based on subject responses. Then the data is transcribed for analysis to obtain research results. Observational data, designed algebraic tasks, and interviews were analyzed based on Swartz and Perkin decision-making. Swartz and Perkin classify thinking skills into three broad categories: creative thinking, retention, and use of information, and critical thinking. Creative thinking skills refer to skills to generate ideas; information retention and use skills refer to skills that promote learning for the understanding and active use of knowledge; while critical thinking skills refer to skills to assess the reasonableness of ideas.

FINDING AND DISCUSSION

The purpose of the observations made by the author on the task about the system of linear equations of two variables is to examine the interaction patterns of tasks in class carried out by the subject, namely how the issue uses subject matter knowledge in giving tasks, knowing students' conceptual knowledge to see whether students can show that a has a solution, both from the universe and from the condition that the System of Linear

Equations of Two Variables has a solution if the universe is determined. The instructional skills and strategies used by the subjects, how they try to identify students' prejudices and learning difficulties, and what the subjects do to correct misconceptions. In this study, researchers used one of 3 questions to be analyzed in depth. The material in this task (how do students understand a system of linear equations of two variables, determine whether a system of linear equations of two variables has a solution if the constant of one of the equations is replaced with a, determine the logical reasons why the system has a solution or not).

"Does the following system of linear equations: $\begin{pmatrix} x - 2y = 10 \\ 3x + 2y = a \end{pmatrix}$ have a solution? Explain your answer".

The researcher observed several things done by the subject and his students, which are presented in the data in Table 2. The researcher's observations included the subject task strategy, interactions with students, and the subject's responses to the student's answers.

Table 2. Exposure to subject and student activity data in the tasks of a Linear Equation System of Two Variables

Linear Equation System of Two Variables Tasks Observations "to determine whether a system of linear equations has a solution or not."	**Subject and student activities**
The subject stands before the class, asking what students know about the Linear Equation System of Two Variables in the compulsory mathematics class. Then ask, "Is it possible that a two-variable system of linear equations can have no solution?" then the subject writes an example $\begin{pmatrix} \boldsymbol{ax - y = 10} \\ \boldsymbol{bx - y = 5} \end{pmatrix}$, "If a = b, what happens to the graph?". Then the subject asked, "Do the children understand what I mean?".	The subject uses his content knowledge to remind students of the requirements for a Linear Equation System of Two Variables. The subject asks questions intending to remind before the subject gives the task. When the subject reminds the students of their prior knowledge or prerequisites, the subject already knows the task strategy.
All students responded to the subject question by answering "yes."	Closed subject questions do not allow students to choose answers and explain student thinking.
The subject wrote down his task "Does the following system of linear equations: $\begin{pmatrix} \boldsymbol{x - 2y = 10} \\ \boldsymbol{3x + 2y = a} \end{pmatrix}$ have a solution? Explain your answer" Tasks allow for multiple solutions and answers	Subjects began using open-ended questions, which allowed students to choose whether or not the system had a solution. Give freedom to students to explain their thoughts and reasons logically.
The subject asks students to discuss with their peers everything they think.	The subject uses the task strategy with discussion.
Student 1 responds orally that the equation has a solution if we replace a with any integer. Students reason and think quickly	The subject accepted the student's answer by saying "good", then asked another student "Is there anything similar to student 1's answer? If so, why?"
Student 2 said, "My answer is the same as student 1 because I tried to enter one example of negative and positive integers".	The subject still accepts student 2, answering by saying well.

Student 3, if any integer replaces a, it means that if any real number replaces it, it also has a solution. The results of solving student problem 3 can be seen in Figure 1.	Students are actively involved in tasks. Students find the set of solutions if $a \in R$ Students reason and think fast, and develop many solutions and strategies
Several other students started scrambling to answer each other, and some said that if any rational number could replace a, the system of equations still had a solution. Whereas	The subject uses content knowledge to explain that the system of equations has a solution or does not depend on the universe of the conversation. Able to be responded to by students at various cognitive levels
Other students explained that if it is not clear what a number is, the system of equations cannot be said to have a solution.	Invite unusual responses and critical thinking
The subject smiled when he heard a student answer that the Linear Equation System of Two Variables had no solution.	The subject asks for explanations from students to get a deeper insight into students thinking. (Pedagogical knowledge)
Another student said, "Sir, I have to see how the graph looks like after it's been drawn, if the two lines are parallel, then it's certain that the system of equations has no solution. If they intersect, then it's certain that the equation has a solution."	Students begin to explore their understanding by remembering the initial knowledge they have. Invite unusual responses and critical thinking.
The subject began to ask, "How do we know if we draw it the two lines will be parallel, intersect, or coincide? Try other students can help their friends!".	The subject wants to know what his students think by using questions to explore students' abilities.
The student smiled and began to realize the answer, "If $\boldsymbol{a} \in \boldsymbol{R}$ and the two equations are not the same then it seems impossible to have a solution" I think like that.	The subject begins understanding his students' thoughts by saying, "Good". The subject understands the misconceptions of his students.
The subject explained maybe what your friends meant "for $\boldsymbol{a} \in \boldsymbol{R}$, (write down the general equation Linear Equation System of Two Variables $\begin{pmatrix} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{pmatrix}$ then the graphs of the two line equations can be parallel or say they do not have a solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the graphs of the two line equations coincide or are said to have many solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, and have one solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. An example of a teacher's notes on problem-solving is shown in Figure 2.	The subject uses content knowledge to clarify whether the system of equations has a solution depending on the coefficient requirements when using the graphical method.
The subject began to ask, "If *a* replace a with any imaginary number, does the system still have a solution?".	The subject again provoked students' understanding which he got in the compulsory mathematics class to answer his questions.
Most students answered "no."	Once again, the students responded enthusiastically to the subject's questions, which expressed their understanding of the Linear Equation System of Two Variables.

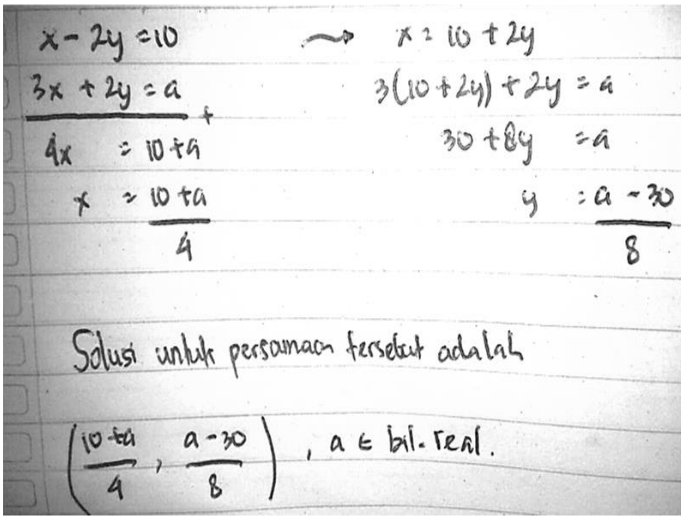

Figure 1. Example of Student Problem Solving

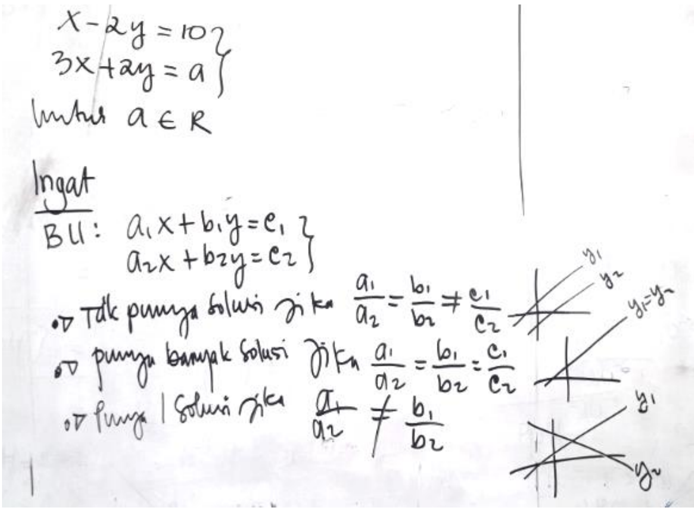

Figure 2. An example of a Teacher's Notes on Problem Solving

The description of the observation of the Linear Equation System of Two Variables tasks was carried out by the researcher to track PCK to find out the teacher's knowledge of the assignment material, the teacher's knowledge of the student's concepts, and the teacher's knowledge of the task strategy. The following describes the subject's decision-making based on excerpts of interviews between the researcher (R) and the subject (S).

Generating Ideas

After the researcher observed the subject's communication with his students in the Linear Equation System of Two Variables material task, the researcher talked about the ideas generated by the subject in designing the task. The subject explained that if these tasks were often given to their students every year, it meant that the subject had used different task characteristics a few years ago so that tasks that were considered very influential on students' conceptual knowledge would be taught again in the next generation to see the responses of their students who differed in every generation. The subject also explained what key concepts students and themselves must have in this task.

R: Where did the idea come from, sir, to make problems in your task today?

S: This is a problem that I give to the previous class every year. Initially, this idea came from modifying tasks in the book. I try again because of my experience every year, and I want to see if this year's student response is the same or better. This year my students responded that if some imaginary number replaced a, would the system still have a solution? I couldn't believe it when they said no. Previously, I wanted it to be easy to use worksheets from MGMP or to receive books from publishers, and then my children told me to open the pages that were to be taught that day. I explained that if no one asked, I would continue to give practice questions in the book. I tell them to come forward; often, only the same children (the smart ones) come forward. I have noticed for a long time how come no one has the same critical answer. I tried the new problem, and it was still the same. I started to ask questions that were quirky or unusual, it seemed that a lot of people were starting to get enthusiastic, and then I tried to introspect myself; during my time as a national instructor, I taught other people to be creative, and then I didn't see myself, so I started to get used to looking for or thinking about questions for my tasks over time it makes me so excited that I forget and rarely use books anymore. Children also like it, and quite a lot that I see have creative potential.

R: It means that this question is often given to students, sir

S: yes

R: Then, what is the key concept that you want to show to your students this time?

S: A solution to a Linear Equation System of Two Variables, for example: using the graphical method, elimination, substitution, or a combination of both, what affects the Linear Equation System of Two Variables to have a solution or not, logical reasons why the Linear Equation System of Two Variables does not have a solution.

Based on the snippet of the interview above, the initial idea of this task came from a task in a student book that was modified or reformed by the subject. The subject gives this routine task to his students every year. The subject felt that there were always different and very interesting student responses every year. The subject also explained the key concepts of the task, namely: the solution of a Linear Equation System of Two Variables, for example: using the graphical method, elimination, substitution, or a combination of both, what influences the Linear Equation System of Two Variables to have a solution or not, the reasons logical why the Linear Equation System of Two Variables does not have a solution. The subject also showed his notes about the possible answers that his students might answer when the subject generated his task idea (Figure 3). In Figure 3, the subject explained to the researcher that he had thought about if his students would think of an answer for the Linear Equation System of Two Variables that has a solution if $\forall a \in R, \forall a \in Q, \forall a \in Z$, and for those that do not have a solution if $\forall a \in \{\}$ and $\forall a \in Imagine$.

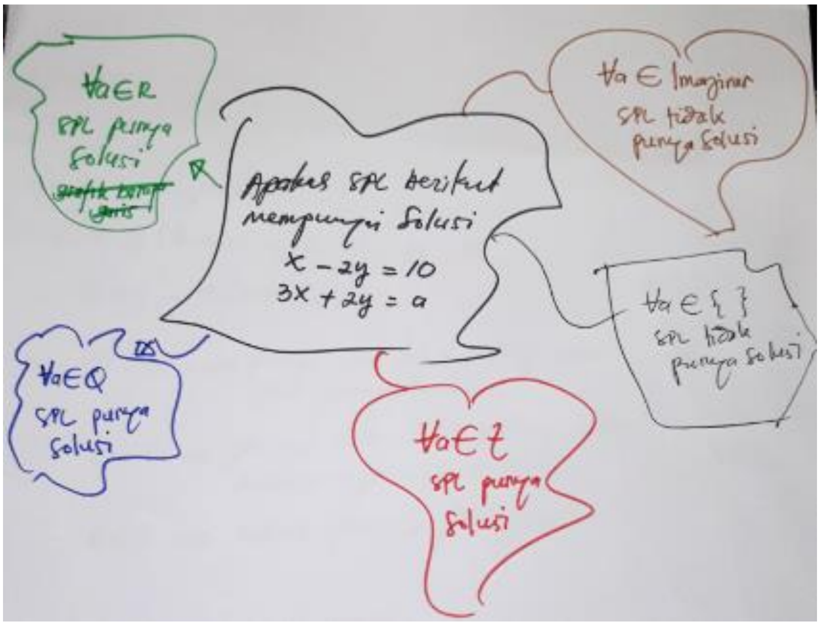

Figure 3. Example of teacher notes in generating task ideas

When the subject explained that one of the subject's considerations for making tasks was the understanding that the children had previously, in fact, the subject wanted to explain about students' prior knowledge, which had to be honed by using investigative questions such as the task. The subject also wanted to change the habits of his students, who were often unsure of their answers and did trial and error as if unsure of understanding the concept. The subject also wants students to be able to analyze their answers and be able to generalize to produce logical arguments. The subject explanation above shows that the subject knows the students' conceptions.

The subject also explained to the researcher that when generating ideas about the task this time, the subject had time to think about other tasks that he might try for his students. Finally, the subject explained the information he needed to produce his current task. This is according to the interview transcript, as below:

> I am thinking of a similar problem that requires the relationship of procedures to concepts. For example, "Write a system of linear equations where (3, -5) is a solution to equation 1 but not a solution to equation 2, and (-1, 7) is a solution to a system of linear equations with two variables." However, it did not work because it took me a long time, let alone a class X student. I also thought about the equation “$\begin{pmatrix} ax - 2y = 10 \\ bx + 2y = c \end{pmatrix}$, does the system have a solution?". So, I direct the children to prove directly. The idea is because I'm used to looking at the Linear Equation System of Two Variables problems and looking for a solution, then I try to replace one of the constants with a, which the universe does not know yet. I also want to know how deep their initial knowledge is about the Linear Equation System of Two Variables.

Based on the interview above, the subject wanted to explain other possibilities he had in mind when generating the idea of the Linear Equation System of Two Variables task this time. The subject has more knowledge about his students' understanding, which can be seen when there is a new task that he has just thought about but does not give students because the time needed will be longer. This task has a high level of difficulty. The purpose of the task ideas made by this subject is to see student understanding, creativity, and critical thinking skills. The subject also paid attention to the students' difficulties when doing the task, so he gave enough time for homework and plenty of space for students to discuss with their friends.

The use of procedural knowledge was very effective (although some students' conceptual insights cannot be guaranteed) in solving subject task problems. Hurrell (2021) argues that procedural learning must be linked to conceptual knowledge to encourage the development of an understanding of concepts that align with what is observed in the subject. Chirove & Ogbonnaya (2021) show that if mathematics teachers apply an alliance of factual knowledge, procedural abilities, and conceptual knowledge, it provides a powerful way to learn algebra. Furthermore, students taught using procedures without understanding concepts are often unsure when or how to use what they know, and such learning is fragile and ineffective. This finding is consistent with Mastuti & Prayitno (2023), findings, which show that the teacher tries to formulate and implement tasks that link procedures to mathematical concepts or present new tasks to facilitate students' mathematical thinking.

In each task, the subject explained that one of the considerations in making the task was the understanding that the children had previously; in fact, the subject wanted to explain students' prior knowledge, which had to be honed by using investigative questions such as the task. The subject also wanted to change the habits of his students, who were often unsure of their answers and did trial and error as if unsure of understanding the concept. The subject also wants students to be able to analyze their answers and be able to generalize to produce logical arguments. The subject explanation above shows that the subject knows the students' conceptions. The subject has more knowledge about his students' understanding, which can be seen when there is a new task that he has just thought about but does not give students because the time needed will be longer. This task has a high level of difficulty. The purpose of the task ideas made by this subject is to see student understanding, creativity, and critical thinking skills. The subject also pays attention to the students' difficulties when doing the task, so sometimes he gives enough time to be used as homework and lots of space for students to discuss with their friends. state that "teachers need to correctly identify students' misconceptions and difficulties and eliminate misconceptions and difficulties by asking probing questions or using appropriate tasks." In generating ideas, subjects modify questions in students' mathematics books and look for questions. -unique questions via the internet. The subject uses an open-task strategy. Wu (2018) asserts that although content knowledge is central to the effectiveness of educators in teaching mathematics, teaching methods play an equally important role in learning. In generating task ideas, the subject has planned tasks that will be done individually or in groups. This aligns with Mastuti, Abdillah, Sehuwaky, et al. (2022), which explain that mathematics teachers' knowledge affects their effectiveness in facilitating learning.

Clarifying Ideas

In this interview session, the subject clarified his ideas about the task to explain the subject's expectations of the task and what the subject had gained from the knowledge possessed by his students.

R: What do you expect from the task (while showing the task notes written by the researcher)?

S: I want them to express what they understand, and I want them to be critical and able to present logical reasons. They can take concrete examples and then generalize those examples.

R: What have you got from your students?

S: The same as my student's comment last year (reminding the researcher of the previous year's observation). Some students replace the value of a with several members of the set of integers, and then they conclude if $a \in Z$ has a solution. Some try with natural numbers and conclude if $a \in N$ has a solution. Some try with natural numbers and conclude if $a \in R$ has a solution. Some immediately say that the system has no solution for various reasons. One of them says that a might not be a number. Some say that it is not clear what number a is. Finally, some never change the value of a, then say that whatever the value of a is, the system of equations has a solution. I like it when I ask if a is any imaginary number, does the system still have a solution, and my students simultaneously answer yes. Even though this has not been proven true, these various answers have become material for an exciting discussion.

R: Is there a critical student chance with the task?

S: Yes, students will think about what to replace a so they have a solution. Alternatively, if you do not have a solution, is it enough to declare a not a number or think, what if a is another set of numbers, such as imaginary? It is also possible that students will think about the graph if they start to determine what member of the set of numbers a is. Could it be that if $a \in R$, the system has no solution, what are the conditions, and more they can think of?

There is no diversity of students' answers that the subject claims are wrong; only reflections made by the subject in discussions and clarifications for justification are new knowledge received by students. The subject explained if the students had the opportunity to be critical in their task this time. The subject explained that students not only think about what number a is so that the system has a solution but also think whether it is enough if a is declared not a number or a member of the empty set to make the system have no solution, what if a is a member of the imaginary set, what is the graph? Is it possible if a is an element of the set of real numbers, but the system still has no solution?

The subject also explained that his task was a modification of the task in the book.

First, the subject wants to describe the Linear Equation System of Two Variables concept. Then, it can apply various effective strategies in determining the set of solutions and checking the correctness of the answers in mathematical problems.

> "If in student books we usually find the problem 'find the solution set of $\begin{pmatrix} x - 2y = 10 \\ 3x + 2y = 5 \end{pmatrix}$?'. Procedurally students can use graphical methods, elimination, substitution, or mixed methods. But if in my task there is a difference. Students must think critically about whether $\begin{pmatrix} x - 2y = 10 \\ 3x + 2y = a \end{pmatrix}$ has a solution. Students must use their reasoning to conclude what the reason is if the system has a solution and what the reason is if it doesn't. My task is also not the same when compared to the teacher in the next class. Actually, we use the same worksheet from MGMP, but I only use it for homework or occasionally when I have assignments out."

Mastuti, Kaliky, & Arman (2022) argue that teachers who understand the subject matter find different ways to represent and be accessible to students. For example, in the task, the subject explains if the students not only think about what number a is so that the system has a solution but also think about whether it is enough if a is declared not a number or a member of the empty set to make the system have no solution, what if a is a member of the set imaginary, how is the graph, is it possible if a is an element of the set of real numbers, but the system still has no solution and so on. This is in line with Kuennen & Beam (2020), which found that to teach mathematics effectively, teachers must have a deep understanding of the mathematical knowledge of the material they teach. The findings on students' conceptual knowledge and misconceptions on subject assignments are consistent with the findings of Timothy et al. (2022), who, in a study focusing on pre-service teacher competency diagnostics on students' misconceptions and difficulties, found that teachers had difficulty analyzing their students' responses. Correctly diagnose misconceptions. To address learning difficulties, such as general arithmetic skills problems, asking probing questions can also reveal the trustworthy source of student misconceptions, not just repeating procedures taught by the teacher.

Assessing the Fairness of Ideas

Researchers conducted interviews to assess the reasonableness of the subject's ideas in generating ideas on tasks. During the interview, the researcher noticed the subject's high belief as a teacher who has high trust in his students.

> I am very confident in my task idea. Implementing this minor disturbance came from the behavior of some children who did not want to respond to this task and the suitability of the children's initial knowledge to give the task I wanted to achieve. Some students suddenly look for a solution to the Linear Equation System of Two Variables by letting the value of a go and suddenly decide that the Linear Equation System of Two Variables has a solution or just following their friends' opinion.

The subject wanted to explain his belief in his task so far. Even the subject knows that several obstacles can affect the integrity of the task. The subject's attention to the success of his students was so detailed that he knew which students received more attention and which did not. In addition to the factors that support teacher tasks, various classroom factors reduce the integrity of the task during the implementation stage. The subject explained several factors that might reduce the integrity of the task, namely student behavior and the suitability of the task. The subject also mentioned that his time and busyness outside of his duties as a teacher were two factors that could have improved the implementation of his tasks.

Another factor observed to impede the integrity of tasks during classroom implementation is the suitability of tasks based on students' previous knowledge. The subject described how students' readiness and comfort hindered tasks in the interview. The subject explained that scaffolding and hindering learning would increase if students lacked the knowledge to engage effectively during tasks.

The subject explained that carrying out tasks like himself was a process and usually could only be given after a few days of teaching. The subject sometimes prefers that his students be subjected to minor distractions, so he can have an excuse to discuss their thoughts outside of class while monitoring their progress effectively. The subject admits that the tasks given to his students often take up much time, but this is appropriate when it relates to the development of his students.

> Apart from making predictions of their answers, the subject always took the time to discuss with other teachers. Many of their responses were almost the same as those of my students. The difference is that some teachers use more abstract examples to prove that the Linear Equation System of Two Variables has a solution. For example, there was an idea if the problem was modified "$\begin{pmatrix} x - y = b \\ 3x + y = a \end{pmatrix}$, under what conditions the Linear Equation System of Two Variables does not have a solution?". As with students, some fellow mathematics teachers also have varying answers. That is what makes me confident in my task.

In the interview above, the subject tried to explain if the source of information being considered was not just mathematics books but also online sources and the results of discussions with fellow friends. The subject also shared the tasks he made, so he knew what kind of response his students would show the next day.

The subject told a lot about the positive impact of his task on the researcher. New things are very inspiring for researchers. Then the researcher asked whether something as good as this could hurt students and the subject began to respond:

> The first time I entered class X this semester and taught exponential power material, I started asking 'What does the function $f(x) = a^x$ mean to you?'. Some students were just silent and did not respond, and some began to respond but not much, for example, it means that the

> number multiplied by itself is x. then I asked again then what is x then? Everyone fell silent. This means that they are not used to being interrogated like that. But now it's different, they are more interactive."

The subject explained that a negative impact might arise at the beginning of the task because students not used to think critically find the teacher's questions so disturbing and make them uncomfortable so; that the subject begins to get used to different tasks so that students get used to it, and their confidence grows.

Mastuti & Prayitno (2023) discussed how students involved with tasks aligned with rich mathematics tasks could explain the reasons behind their solutions. They also demonstrated high-level cognitive processes at the end of their evaluations. The subject assessed whether the ideas generated in the previous task were acceptable to the students or whether the teacher's questions bothered them. There is a relationship between teacher questions on tasks and student performance on high ratings because questioning on tasks reflects the expected level of student thinking in class (Stehle & Peters-Burton, 2019). The strategy of assigning open and accessible subjects to convey arguments gets a positive response from students. The subject explained that if students interact with mathematics problems continuously, they have high self-confidence when conveying their answers because the subject often gives positive responses to student answers. In addition, students are allowed to issue their ideas and answers so that the subject can easily diagnose students' misconceptions and difficulties. Overall, the subject's knowledge of the task strategy is considered adequate. The subject uses dialogue to help students develop thinking habits, analyzing, and verbalizing students and making it possible to actively discuss students' ways of thinking with each other when they try to solve problems. Researchers increasingly consider the quality of dialogue and class discussion essential elements in this task (Ozan & Kıncal, 2018).

CONCLUSION AND SUGGESTION

Based on the results and discussion, the following findings are obtained: 1) The teacher utilizes knowledge of each task material used to make decisions in developing his tasks; 2) The teacher makes use of knowledge about students' concepts and misconceptions, which are used to make decisions in developing their tasks; 3) The teacher utilizes knowledge of task strategies used to make decisions in developing tasks; 4) The teacher utilizes the knowledge of the task curriculum. Based on the decision-making process, **at the stage of generating ideas**, the subject is more dominant in considering students' abilities and exploring students' critical thinking processes, namely by reforming tasks with open and creative questions in building students' ideas. **At the stage of clarifying ideas**, the dominant subject considers the students' abilities towards the concepts students have and clarifies student misconceptions that may arise during tasks. The subject knows what his students need. The subject knows his students' initial or previous knowledge, so this becomes subject to consideration in developing his tasks. The subject also considers students' difficulties in each task. **At the stage of assessing the fairness of ideas**, the subject's assessment is based on the subject's belief in himself and his students.

Further research suggests that decision-making can be observed from the student's point of view. This relates to students' thinking processes and focuses on analyzing students' arguments in each task. The instruments will also be developed in other scientific studies, such as geometry or arithmetic.

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